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TITLE: UNCERTAINTIES ASSOCIATED WITH INERTIAL-FUSION IGNITION

AUTHOR(S): Gene H. McCall

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Uncertainties associated with inertial-fusion ignition

Gene H. McCall

Applied Theoretical Physics Division, Los Alamos National Laboratory
Los Alamos, New Mexico 87545

Introduction

At present the energy required to drive an inertial fusion implosion is not well-defined. In the past the energy predicted to produce a breakeven yield, where the thermonuclear energy release is equal to the driver energy, has varied from 10^3 to 10^6 joules. The reason for this large variation is partly that experiment and theory have now defined previously poorly understood physical effects and partly that more conservative estimates are now used for those effects which are not yet understood.

In this paper I will attempt to estimate a "worst case" driving energy which is derived from analytic and computer calculations. It will be shown that the uncertainty can be reduced by a factor of 10 to 100 if certain physical effects are understood. That is not to say that the energy requirement can necessarily be reduced below that of the "worst case," but it is possible to reduce the uncertainty associated with ignition energy. With laser costs in the $0.5-1. billion per MJ range, it can be seen that such an exercise is worthwhile.

Because of the tutorial nature of this paper and because of space limitations, it will be impossible to present detailed justification for all of the results, but an attempt will be made to strike a balance between rigor and intuitive understanding.

Basic principles

An inertial fusion implosion system, typically, consists of a thermonuclear fuel (a deuterium-tritium mixture) surrounded by a container, which serves as a piston to compress the fuel, and a low-density ablator as shown in Figure 1. Energy from a source, a laser for example, is absorbed by the ablator, and hot ablator material expands into the surrounding vacuum. Momentum is conserved in this system, ignoring the laser radiation pressure, and an inward-directed reaction force is produced which drives the interior of the target inward. The DT fuel is compressed and, therefore, heated by PdV work to a point where thermonuclear reactions are produced. If the range of alpha particles produced by the reaction $D+T \rightarrow \alpha+n + 17.5$ MeV is shorter than the radius of the compressed fuel region, the thermonuclear reactions themselves produce additional fuel heating which increases the reaction rate. The point at which this rapid increase, or bootstrapping, of the fuel energy occurs is referred to as "ignition."

A number of physical effects interact to produce ignition. The details of the process by which energy is absorbed by the ablator determine the conditions in the blowoff plasma, which, in turn, determine the momentum and energy transfer to the target interior. Also, the sudden heating of the ablator can produce shock waves which change the initial condition of the interior, and high energy electrons, such as those produced by long wavelength lasers, can heat the target interior before compression occurs, thus, making the fuel more difficult to compress. The energy transferred by these non-isentropic processes is usually identified as "preheat." The ignition process depends upon volume deposition of alpha particles, and particles can be lost from the surface of the fuel. Therefore, one is required to maintain as large a volume to surface ratio as possible, which implies that the fuel region must remain nearly spherical, and that symmetry of the implosion is important. Too, instability at the inner surface of the pusher can eject pusher material into the fuel, and , effectively, remove fuel from the ignition region. Finally, the fuel must

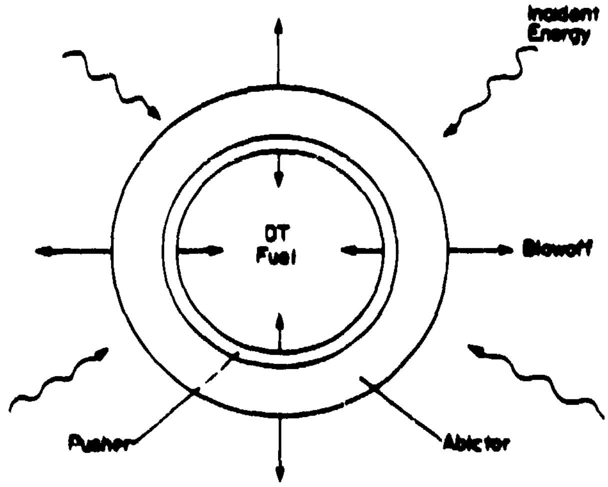

Figure 1. Schematic representation of inertial fusion principles.

remain compressed and heated long enough for ignition and burn to occur.

In the discussion below I will examine these effects and will attempt to place limits on them. The question of thermonuclear burn will not be discussed. The emphasis in inertial fusion is on a "demonstration of principle" which, I believe, is satisfied by a demonstration of ignition. Therefore, the emphasis in the discussion which follows is placed on the approach to ignition conditions in the fuel.

Absorption of driver energy

The absorption of energy by an ablator has been measured and calculated for laser drivers in the wavelength range $10\ \mu m > \lambda > 0.25\ \mu m$.[1] The absorption coefficient varies from 0.25 at 10 μm to 0.9 at 0.25 μm. The dominant absorption process in the intensity range of interest (10^{13}-10^{16} w/cm^2) at wavelengths of 1-10 μm is resonant absorption, which produces high energy electrons.[2] At shorter wavelengths, inverse Bremsstrahlung, or classical absorption, appears to be the important mechanism.[3] The uncertainty in the experimental values of the absorption coefficient is $\sim \pm$ 15 percent.

Hydrodynamic efficiency and momentum transfer

The process of momentum transfer from an expanding gas can be characterized in terms of the "rocket model" used as a momentum conservation example in undergraduate physics courses.[4] Consider a rocket of initial mass, M_o which has an exhaust velocity, C_s and a mass ablation rate. $\dot{m}$, starting from rest. If V is the rocket velocity at time, t, when the remaining mass is $M = M_o - \dot{m}t$, the momentum equation can be written as

$$c_s \frac{dM}{dt} = M \frac{dv}{dt} . \tag{1}$$

This equation can be solved easily to give:

$$V = C_s \ln \frac{M_o}{M} . \tag{2}$$

The efficiency of energy transfer, η, is the ratio of the kinetic energy of the rocket to the total energy released which is:

$$\eta = \frac{\ln \frac{M_o}{M}}{\left(\frac{M_o}{M}\right) - 1} . \tag{3}$$

A useful quantity for determining desirable ablator conditions is the momentum per unit energy, P/E, which is transferred to the rocket. Intuitively, one can see that momentum is an important quantity in that it ultimately requires a force to compress the target, and force is just the time-rate-of-change of momentum. From Eq. (2) and Eq. (3) one obtains:

$$P/E = \frac{2\eta}{V} = \frac{2 \ln \frac{M_o}{M}}{\left(\frac{M_o}{M} - 1\right) C_s} . \tag{4}$$

Thus, it can be seen that momentum is transferred more efficiently if a large mass is ablated at low velocity than if a small mass is ablated at high velocity. Of course the velocity cannot be made vanishingly small for a number of reasons. The size of the target determines its mass and, and thus, the energy requirement so the acceleration distance must be made small, and high velocities are required. The mass of pusher which can participate in the implosion is determined by the distance a sound wave can travel during the burn time, and the losses in the system, such as thermal conduction and radiation losses, must be overcome by power, or force times velocity, applied to the system. Therefore, a minimum velocity is required. It is unlikely that ignition can be produced at a velocity below 10^7 cm/s and, possibly, not below 2×10^7 cm/s.

Figure 2 shows efficiency and velocity as a function of remaining mass. Also shown are data obtained at the Naval Research Laboratory from the acceleration of a laser-heated target.[5] The discrepancy between theory and experiment is ascribed by the NRL group to two-dimensional effects. For maximum efficiency, then, one should choose a blowoff velocity which is 1/1.6, or 0.63, of the desired implosion velocity and should ablate, approximately 80 percent of the target mass.

Also shown in Figure 2 is the result of assuming that the target is driven by an isothermal rarefaction. In such a rarefaction the density and velocity of the ablated material is given by:

$$\rho = \rho_o \exp\left(-\frac{x}{C_s t} - 1\right)$$
$$v = C_s + \frac{x}{t} \qquad (5)$$

Material which is accelerated beyond the sound velocity, C_s, with respect to the target surface is ineffective in transferring momentum to the target, but energy is carried by electrons to this material. Also, approximately one-half of the energy is contained in the electrons. The curve of Figure 2 assumes that all of the electron internal energy is lost, but in some cases this energy can be given to the blowoff kinetic energy resulting in a factor of 2 increase in efficiency. It is interesting that the measured efficiency lies near this value.

It seems prudent, therefore, to assign an uncertainty in energy requirement of a factor of 2 to the hydrodynamic efficiency and absorption, combined.

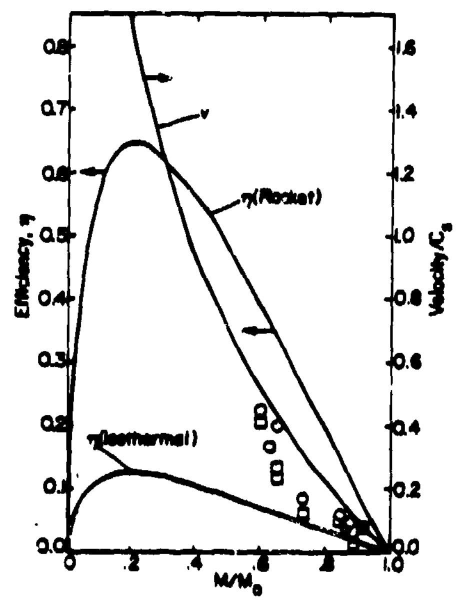

Figure 2. Energy coupling efficiency and implosion velocity as functions of the mass remaining in the target.
(O) - measured velocity (NRL)
(□) - measured efficiency (NRL)

Isentropic implosion

Before discussing the other parameters of interest it is instructive to develop a simple analytic theory of implosion. First, though, a few words should be said about why one implodes a target at all. It was mentioned earlier that one is required to stop in the fuel alpha particles which are released by the thermonuclear reactions. Therefore, a minimum density-length product, or ρR, is required. Also a minimum temperature is required for the initial production of these reactions at a rate higher than the rate at which energy is lost from the system. For an ideal gas, such as DT, the specific internal energy in the fuel is a function of temperature alone, and the total energy, E, in the fuel is

$$E \sim MT \qquad (6)$$

where, M, is the fuel mass and, T is the temperature. If the density is ρ and the radius of the sphere is R, the mass is:

$$M = \frac{4\pi}{3} \rho R^3 \qquad (7)$$

Then,

$$E \sim \rho R^3 T, \qquad (8)$$

$$E \sim \frac{(\rho R)^3 T}{\rho^2} . \qquad (9)$$

It can be seen that if (ρR) and T are determined by ignition criteria, the fuel energy and, hence, the total system energy can be made smaller by increasing the fuel density.

After observing that one reduces the required energy as $1/\rho^2$, one can ask if it is possible to determine the most efficient way of doing this. The change in the specific internal energy, e, which results from heating or compressing the fuel gas can be written in the usual way as

$$de = T\,dS - p\,d\,(1/\rho) \tag{10}$$

where dS is the change in entropy, p is the pressure, ρ is the density ($\rho = 1/V$), and T is the temperature. A compression increases ρ so that $d(1/\rho) < 0$. It is apparent, therefore, that if one wishes to compress the fuel with the smallest energy investment, then one should design a system such that:

$$dS = 0 \quad . \tag{11}$$

Assuming that it is possible to do this, one can write the equation of state of the fuel as

$$pV^{\gamma} = \text{const.} = p_o V_o^{\gamma} \quad , \tag{12}$$

where γ is the ratio of the specific heat at constant pressure to the specific heat at constant volume. For an ideal monatomic gas, such as DT, $\gamma = 5/3$. Also for an ideal gas, the internal energy can be written, using Eq. (10), Eq. (11), and Eq. (12), as:

$$p = (\gamma-1)e\rho \tag{13}$$

Integrating Eq. (10) with Eq. (13) substituted for p as ρ increases from ρ_o to ρ and assuming that $\rho >> \rho_o$, one obtains

$$e = e_o \left(\frac{\rho}{\rho_o}\right)^{\gamma-1} \tag{14}$$

where e_o is the initial internal energy and ρ_o is the initial fuel density. Eq. (14) defines the relationship between energy, or temperature, and compression in an isentropic process.

Eq. (14) expresses the internal energy of the fuel in terms of its compression, and Eq. (13) relates the input energy to the initial implosion kinetic energy. One now requires a dynamic relationship between the two equations to describe the entire system. A target such as that of Figure 1 converts pusher kinetic energy to fuel internal energy with an efficiency ranging from 0.25 to 0.5. This efficiency can be calculated from pressure balance requirements, but the essential features of the implosion can be displayed by assuming this efficiency to be a constant, a. Thus, a fraction, a, of the kinetic energy lost by the pusher appears as internal energy in the fuel, and one can write an energy conservation equation as:

$$\frac{1}{2}\, a\, M_p\, (v_o^2 - v^2) = E_o \left(\frac{\rho}{\rho_o}\right)^{\gamma-1} - E_o \quad . \tag{15}$$

Assuming the fuel to be contained in a sphere of radius, R, one can easily derive a first-order differential equation of the form:

$$\left(R\frac{dR}{dt}\right)^2 + AR^2 = B \quad . \tag{16}$$

The minimum radius, R_m, of the implosion is determined from energy balance:

$$\left(\frac{\rho}{\rho_o}\right) = \left(\frac{a}{2E_o} M_p v_o^2\right)^{\frac{1}{\gamma-1}} \tag{17}$$

where V_o is the initial implosion velocity and M_p is the mass of the pusher. Eq. (7) can be used to determine R_m from Eq. (17). If R_o is the initial fuel radius and $R_o >> R_m$, the solution of Eq. (16) is[6].

$$R = R_m \left[1 + C^{\frac{2}{3}} (t'-1)^2\right]^{\frac{1}{2}} \quad , \tag{18}$$

where

$$C = (R_o/R_m)^3 \quad , \tag{19}$$

$$t' = \frac{v_o}{R_o} t \quad ,$$

and

$$\gamma = \frac{5}{3} \quad .$$

Figure 3 shows a comparison between the results of Eq. (18) and a computer simulation of an implosion. The computer simulation includes radiation losses. It can be seen that Eq. (18) is an accurate description of this class of implosion.

Next, conditions for ignition must be determined. As a criterion for defining ignition, one can specify that the thermonuclear energy deposited in the fuel be equal to the energy deposited by the compression. Thus, the thermonuclear process will double the fuel temperature and bootstrapping can be observed. Using Eq. (18) one can determine the confinement time, Δt, and insert it into the well-known expression for yield, Y:

$$Y = n_D \, n_T \, \overline{\sigma V} \, \Delta t \tag{20}$$

where n_D and n_T are the deuterium and tritium ion densities, $\overline{\sigma V}$ is the thermonuclear reaction rate, and V is the volume. Because of the strong dependence of $\overline{\sigma V}$ on temperature ($\sim T^4$) in the region of interest, one can allow the density to change only 10 percent thus, Δt is determined. The (ρR) required is determined from the alpha particle range ($\rho R_\alpha = 0.3T$ g/cm^2) in a plasma. Because most of the mass is contained near the outer edge of a sphere, the effective ρR is, actually, only 0.75 of the actual ρR[7]. The calculation will not be displayed explicitly here because of space limitations, but a reasonable choice of implosion system parameters gives:

$$\rho R = 0.5 \text{ g/cm}^2 \tag{21}$$

$$T = 4 \text{ keV}$$

as an ignition point.

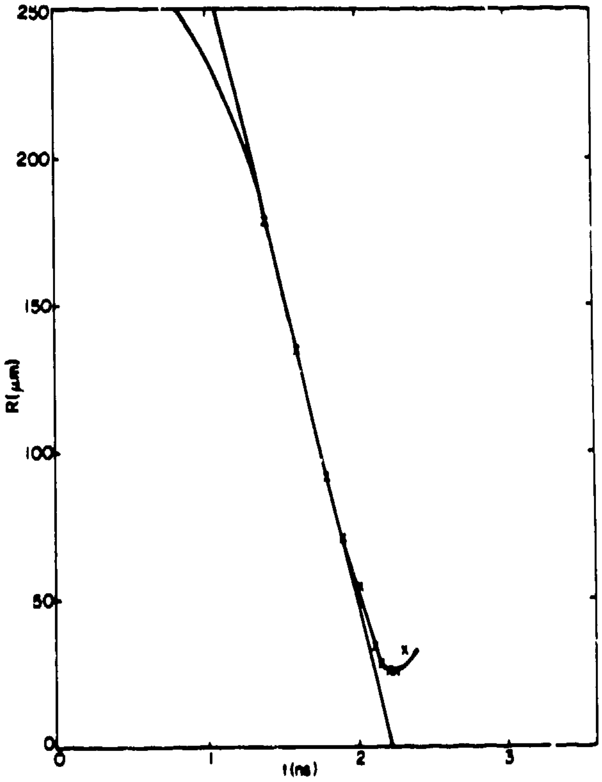

Figure 3. Fuel radius as a function of time for a typical implosion. Solid line is the analytic model. Points are from a computer simulation.

Preheat

Figure 4 shows two isentropes on a T-ρ diagram. The upper isentrope passes through the ignition region of Eq. (21) and the lower isentrope passes through the initial point for DT at 0.2 g/cm^3. The dotted line in between is the Fermi temperature of DT as a function of density. It may be possible to improve efficiency in an implosion by remaining below the Fermi temperature, but very high compression ($\sim 10^4$) would be required for ignition. Therefore, it has been assumed that the ignition conditions should be reached at a more modest compression of 10^3.

The transition from the lower to the upper isentrope is made almost automatically in an inertial fusion target as the result of shock, electron, and hard x-ray heating. The energy, e_o, of Eq. (14) is just that required to establish the isentrope if the energy is added at zero time while the fuel density has its initial value. This energy has been defined as the "equivalent initial preheat."[8] If, however, one can compress the fuel without preheating it by a compression factor, C_p, reference to Eq. (14) immediately shows that the ignition isentrope is then reached by the addition of an energy.

$$e_p = e_o C_p^{\frac{2}{3}} \quad . \tag{22}$$

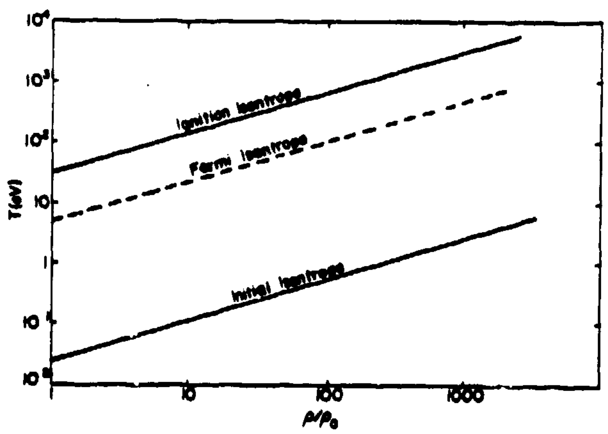

Figure 4. Isentropic implosion trajectories. Fuel temperature as a function of fuel density.

Thus, it is possible to make the system more tolerant to preheat if an initial, cold compression can be achieved. For long wavelength lasers, such as CO_2, I believe that this pre-compression is essential and that shaped pulses will be required to achieve it. A pre-compression of 30, for example, allows 10 times as much preheat. If preheating is by fast electrons, a factor of 10 increase in preheat is produced by reducing the shielding thickness by a factor of 2, which gives a mass (and energy) reduction in the target of 4 to 8. Even with this reduction it is apparent that a long-wavelength laser will require ∿ 2 times the energy of a short-wavelength laser for ignition. Also, no experimental investigation of cold compression has been undertaken, and it is necessary to assign a large uncertainty to the gains which may be achieved. In the absence of such an investigation it seems reasonable to assign a factor of 2 uncertainty to short-wavelength preheat effects and a factor of 4 to long-wavelength preheat effects.

Stability

When a dense fluid, such as the pusher of Figure 1, is accelerated by a fluid of lower density, such as DT gas, the interface between the two fluids is unstable for small perturbations. This is the Rayleigh-Taylor instability,[9] and it can occur during deceleration of the pusher by the fuel. Rather than discussing the physical process itself, an attempt will be made to estimate its effect on an implosion.

If a perturbation of the fuel-pusher interface grows rapidly enough, it is possible that a fraction of the pusher mass will penetrate the fuel region. As a worst case consider the possibility that pieces of the pusher are unaffected by the pressure of the fuel and that they continue to move at the initial velocity rather than following the trajectory of Eq (18). If the pusher shell is, for the most part, intact, the density of the fuel can be obtained from Eq. (18), but the portion of the fuel penetrated by pusher material will not be available for ignition. Thus, ρ is determined by Eq. (18) and ρR is determined from the radius of the "free-fall" line of Figure 3. That is,

$$R = R_o \, (1-t') \quad . \tag{23}$$

Therefore,

$$(\rho R)_f = \frac{(\rho R)_m \, C^{\frac{1}{3}} \, (1-t')}{\left[1 + C^{\frac{2}{3}} \, (t'-1)^2\right]^{3/2}} \tag{24}$$

where $(\rho R)_m$ is the value of ρR achieved at maximum compression in the ideal case. By differentiating Eq. (24) and setting the derivative equal to zero, one finds that the maximum value of $(\rho R)_f$ is achieved at a time:

$$t_f' = 1 - \frac{1}{\sqrt{2}\, C^{1/3}} \tag{25}$$

and that the maximum value is 0.38 $(\rho R)_m$. The density at this time is 0.54 of the maximum density.

Reference to Eq. (8) shows immediately that the energy in the implosion must be increased to satisfy the conditions of Eq. (21). The additional energy required depends on assumptions as to how implosion conditions can be changed. Noting that the ideal target will continue to implode to a temperature $(.54)^{-2/3}$, or 1.5, times that at t_i', and that the ρR must increase to a value 1/0.38 times that of the ideal case, the ratio of energy required for free-fall ignition, E_f, to that required for the ideal case, E_i, is, using Eq. (9):

$$\frac{E_f}{E_i} = \left(\frac{1}{0.38}\right)^3 (1.5) \frac{\rho_1^2}{\rho_2^2} = 27 \left(\frac{\rho_1}{\rho_2}\right)^2 \quad (26)$$

where ρ_1 is the maximum density in the ideal case and ρ_2 is the density to which the free-fall case would implode if no instability occurred. If it is assumed that the ideal compression factor cannot be increased and that the increased ρR must be obtained by adding fuel to the system, then $\rho_1 = \rho_2$ and 27 times as much energy is required. If, however, the compression factor can be increased, say by increasing the velocity, then $\rho_2 = (1.5)^{3/2}\ \rho_1$ and $E_f/E_i = 8$. Therefore, the energy required for the "worst case" of instability is 8 to 27 times that required in the ideal case.

Consider now an implosion driven by a shaped pulse such that the implosion velocity remains constant until ignition time. The energy required to accomplish this can be obtained by integrating the power (Force X velocity), and it is found that the fuel energy is the same as in the case described by Eq. (18), as it should be for an isentropic process. At ignition time, however, the pusher contains an energy $M_p\, v_o^2/2$, which was the total initial energy for the case of Eq. (18). Therefore, the "worst case" energy is twice that of the ideal case. It can be seen, then, that pulse shaping reduces the uncertainty in required energy by a factor of, approximately, 10. The power requirements may be extreme, however.

Symmetry

The discussion of stability addressed the problem of a uniform loss of fuel at the inner surface of the pusher. If the equator of the pusher is driven at a velocity slightly different from the polar velocity, the implosion does not remain spherical and one finds a reduction in ρR as the result of this asymmetry.

As an example of the effect, it will be assumed that the fuel implodes as a prolate ellipsoid, and it will be assumed that the worst case of stability exists. That is, the decrease in ρR as a result of asymmetry should be compared to the ρR the free-fall line. Then one assumes that the equator implodes at a velocity, $v_1 = v_o + \Delta v/v_o$, and the pole implodes at a velocity, $v_2 = v_o - \Delta v/v_o$, where v_o is the spherical implosion velocity, the ratio of the elliptical ρR, $(\rho R)_e$, to the spherical ρR, $(\rho R)_f$, can be written as:

$$\frac{(\rho R)_e}{(\rho R)_f} = \frac{\left(1 - \frac{v_1}{v_o} t'\right)\left(1 - \frac{v_2}{v_o} t'\right)}{(1-t')^2} . \quad (27)$$

Figure 5 shows this ratio as a function of $\Delta v/v$ for several compression ratios. The line at 0.8 denotes the point at which the energy requirement doubles. Thus, for a factor of 2 increase in energy for $C = 10^3$, a total velocity spread $2\Delta v/v = 0.1$ is acceptable while for a compression of 10^4, only a 4 percent spread is allowable. Assuming that 10 percent is achievable, the compression is limited to 10^3 and the energy uncertainty is a factor of 2.

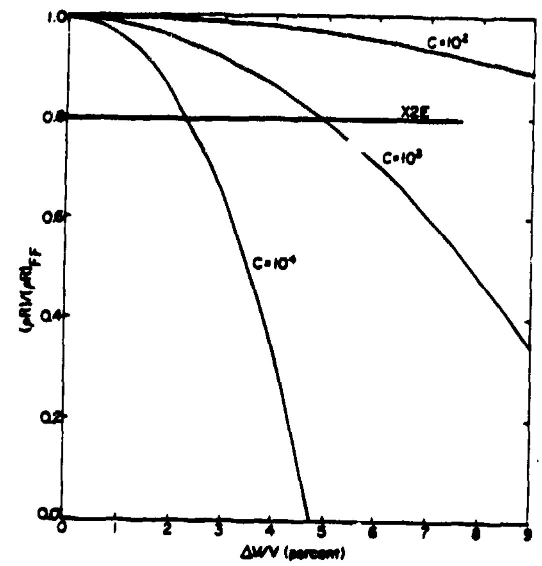

Figure 5. Ratio of fuel (ρR) to ρR of the free-fall line as a function of the velocity asymmetry for several values of compression, C. The line X2E indicates the ρR for which input energy must be doubled.

Summary

It is instructive to combine these uncertainties to produce an overall "worst case." Table I shows a summary of the uncertainties derived from effects described above. The equation-of-state problem and "other losses" was not discussed, but a similar analysis shows the effects to be small, probably no more than ± 20 percent. A typical target consisting of 13 μg of DT with a gold pusher requires, approximately, 10 kJ in the fuel-pusher system to ignite at a compression factor of 10^3. A hydrodynamic efficiency of 15 percent gives 67 kJ as the driving energy required for the ideal case, and the uncertainties shown in Table I produce the energy requirements shown in Table II. It should be noted that the quoted energies are absorbed energies and should be divided by an absorption coefficient to give the driver energy required. The quantities of Table I and Table II are phrased in terms of a laser driver, but the application to other drivers is straightforward.

Table I. Summary of uncertainty factors

	Shaped Pulse		Impulsive Driver	
Effect	Long Wavelength	Short Wavelength	Long Wavelength	Short Wavelength
Absorption, equation of state Hydro. efficiency, other losses	2	2	2	2
Preheat	4	2	10	2
Stability	2	2	4	25
Symmetry	2	2	2	2

Table II. Worst case ignition threshhold

Driver	Absorbed ignition energy (MJ)
Ideal implosion	0.07
Short wavelength - shaped pulse	1
Long wavelength - shaped pulse	2
Short wavelength - impulsive	10
Long wavelength - impulsive	20

The energies quoted are "worst case," and the arguments used to produce these values are not rigorous in all cases. If the ideal case at $C = 10^3$ can be achieved, the required energy is ∿ 70 kJ or, perhaps, even slightly lower. The real point of this discussion is that the absence of knowledge about several physical effects results in large uncertainties in energy requirements. The conclusions to be drawn from this analysis are that: 1) shaped pulses are useful for all drivers and essential for long-wavelength lasers, and

2) most of the theoretical and experimental effort in the ICF program should be devoted to the study of preheat, symmetry and stability.

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